Teleportation of arbitrary n-qudit state with multipartite entanglement

Zhan-jun Zhang[a,b]

[a] Department of Physics and Center for Quantum Information Science,
National Cheng Kung University, Tainan 70101, Taiwan

[b] School of Physics & Material Science, Anhui University, Hefei 230039, China

We propose a protocol $\mathcal{D}_n$ for faithfully teleporting an arbitrary n-qudit state with the tensor product state (TPS) of n generalized Bell states (GBSs) as the quantum channel. We also put forward explicit protocol $\mathcal{D}_n'$ and $\mathcal{D}_n''$ for faithfully teleporting an arbitrary n-qudit state with two classes of $2n$-qudit GESs as the quantum channel, where the GESs are a kind of genuine entangled states we construct and can not be reducible to the TPS of n GBSs.

I. Introduction

No-cloning theorem forbids a perfect copy of an arbitrary unknown quantum state. How to interchange different resources has ever been a question in quantum computation and quantum information. In 1993, Bennett et al.[1] first presented a quantum teleportation scheme $\mathcal{T}_0$. In the scheme, an arbitrary unknown quantum state in Alice's qubit can be teleported to a distant qubit B with the aid of Einstein-Podolsky-Rosen (EPR) pair. Suppose Alice has a qubit x in an arbitrary unknown normalized state

$$|\Lambda\rangle_x = \alpha|0\rangle_x + \beta|1\rangle_x, \tag{1}$$

where α and β are complex. Alice and a remote Bob share an EPR pair (a, b), say, in the state

$$|\Psi_0\rangle_{ab} = \frac{1}{\sqrt{2}}\sum_{j=0}^{1}(|j\rangle|j\rangle)_{ab}. \tag{2}$$

This teleportaion between Alice and Bob can be seen intuitively from the following equation,

$$|\Lambda\rangle_x|\Psi_0\rangle_{ab} \ = \ \sum_{i=0}^{3}|\Psi_i\rangle_{ax}\sigma_b^{(i)}|\Lambda\rangle_b, \tag{3}$$

where $|\Psi_i\rangle_{ab} = \sigma_b^{(i)}|\Psi_0\rangle_{ab}$, $\sigma^{(0)} = |1\rangle\langle 1| + |0\rangle\langle 0|$, $\sigma^{(1)} = |0\rangle\langle 1| + |1\rangle\langle 0|$, $\sigma^{(2)} = |0\rangle\langle 1| - |1\rangle\langle 0|$ and $\sigma^{(3)} = |1\rangle\langle 1| - |0\rangle\langle 0|$. Bennett et al's work showed in essence the interchangeability of different quantum resources[2].

The teleportation of multi-qubit teleportation has been studied by Lee et al[3] and Yang et al[4]. Suppose that the arbitrary $n(n\geq 2)$-qubit state Alice wants to teleport to Bob is written as

$$|\Lambda\rangle_{x_1x_2\ldots x_n} = \sum_{m_N=0}^{1}\cdots\sum_{m_2=0}^{1}\sum_{m_1=0}^{1}C_{m_1m_2\ldots m_N}|m_1\rangle_{x_1}|m_2\rangle_{x_2}\ldots|m_n\rangle_{x_n}, \tag{4}$$

where C's are complex coefficients and $|\Lambda\rangle_{x_1x_2\ldots x_n}$ is assumed to be normalized. Alice and Bob share in advance N same Bell states, say, $|\Psi_0\rangle_{a_nb_n}\otimes\cdots\otimes|\Psi_0\rangle_{a_2b_2}\otimes|\Psi_0\rangle_{a_1b_1}$. The n qubits a_1, a_2, $\cdots$, $a_{n-1}$and a_n is in Alice's site. The n qubits b_1, b_2, $\cdots$, $b_{n-1}$and b_n in Bob's site are used to "receive" the teleported state from Alice. Hence, the initial joint state is

$$|\Lambda\rangle_{x_1x_2\ldots x_n}\otimes|\Psi_0\rangle_{a_nb_n}\otimes\ldots\otimes|\Psi_0\rangle_{a_2b_2}\otimes|\Psi_0\rangle_{a_1b_1}. \tag{5}$$

It can be rewritten as[5]

$$\frac{1}{2^n}\sum_{i=1}^{2^n}|\Psi_{i_n}\rangle_{a_nx_n}|\Psi_{i_{n-1}}\rangle_{a_{n-1}x_{n-1}}\cdots|\Psi_{i_1}\rangle_{a_1x_1}U_{i_ni_{n-1}\cdots i_1;b_nb_{n-1}\cdots b_2b_1}|\Lambda\rangle_{b_1b_2\cdots b_{n-1}b_n}. \tag{6}$$

If Alice performs n Bell-state measurements on the qubit pairs (a_n, x_n), ..., (a_1, x_1) and publishes a $2n$-bit classical message corresponding to her measurement outcomes on the qubit pairs, then conditioned on Alice's information, Bob can recover the arbitrary state $|\Lambda\rangle$ by performing at most $2n$ single-qubit operations. To our knowledge, as for as the multipartite quantum state teleportation is concerned, only protocols for n-qubit state teleportation are proposed[3-9], and so far there does not exist any protocol for teleporting an arbitrary n-qudit state though teleportation for one-qudit state has ever been studied by Zubairy[10], Stenholm and Bardroff[11], and Roa et al[12]. In this paper we will extend such sutdies. We will directly consider the general case of teleporting an arbitrary n-qudit state with the tensor product state (TPS) of n generalized Bell states (GBSs) as the quantum channel.

On the other hand, recently Rigolin[6] has proposed a protocol for teleporting an arbitrary two-qubit state with a four-particle generalized Bell state as a genuine quantum teleportation channel and a four-particle joint measurement. However, the multipartite state in the Rigolin's protocol is just a tensor product state of two Bell states in essence, not a genuine multipartite entangled state[7]. As a consequence, the Rigolins protocol[6] is equivalent to the Yang-Guo protocol[4] for teleporting an arbitrary multipartite state in principle. Very recently, Yeo and Chua[8] have presented an explicit protocol for faithfully teleporting an arbitray two-qubit state via a genuine 4-qubit entangled state they constructed. They think it is an important consideration because the four-qubit entangled state, in addition to two Bell states, could be a likely candidate for the genuine four-partite analogue to a Bell state. Soon later, Cheng, Zhu and Guo[9] presented a general form of genuine multipartite entangled quantum channels for arbitrary qubit-state teleportation. In this paper we will present an explicit protocol for faithfully teleporting an arbitrary n-qudit state with two classes of $2n$-qudit GESs, where GES is referred to as a kind of genuine entangled states we construct and can not be reducible to the TPS of n GBSs.

This paper is organized as follows: In section II we will propose a faithful teleportation protocol $\mathcal{D}_n$ of multipartite n-qudit state with the TPS of n GBSs as the quantum channel. In section III we will present an explicit protocols $\mathcal{D}'_n$ and $\mathcal{D}''_n$ for faithfully teleporting an arbitrary multipartite n-qudit state with two classes of GESs. A brief summary is given in section IV.

II. Protocol $\mathcal{D}_n$ for teleporting arbitrary n-qudit state using TPS of n GBSs

Teleportation for one-qudit state has ever been studied by Zubairy[10], Stenholm and Bardroff[11], and Roa et al[12]. However, so far there does not exist any protocol concerning the teleportation of an arbitrary n-qudit state. In this section we will focus on this issue and propose a faithful teleportation protocol $\mathcal{D}_n$ of multipartite n-qudit state with the TPS of n GBSs as the quantum channel.

Suppose Alice has n qudits $\{X_1, X_2, \cdots, X_n\}$ in the state of

$$|\Lambda\rangle_{X_1X_2\cdots X_n} = \sum_{j_1=0}^{d-1}\sum_{j_2=0}^{d-1}\cdots\sum_{j_n=0}^{d-1} C_{j_1j_2\cdots j_n}|j_1j_2\cdots j_n\rangle_{X_1X_2\cdots X_n}, \tag{7}$$

where C's are complex coefficients and $|\Lambda\rangle_{X_1X_2\cdots X_n}$ is assumed to be normalized. Moreover, Alice and Bob share in advance n generalized Bell states (GBSs) in the form

$$|\Theta_{0000\cdots00}\rangle_{A_1B_1A_2B_2\cdots A_nB_n} = |\Phi_{00}\rangle_{A_1B_1}|\Phi_{00}\rangle_{A_2B_2}\cdots|\Phi_{00}\rangle_{A_nB_n}, \tag{8}$$

where

$$|\Phi_{00}\rangle = \sum_{j=0}^{d-1} |jj\rangle/\sqrt{d}. \tag{9}$$

Alice has the n qudits $\{A_1, A_2, \cdots, A_n\}$ while Bob the n qudits $\{B_1, B_2, \cdots, B_n\}$. Hence the state of the $3n$-qudit system is

$$|\Gamma\rangle_{X_1X_2\cdots X_nA_1B_1A_2B_2\cdots A_nB_n} = |\Lambda\rangle_{X_1X_2\cdots X_n}|\Phi_{00}\rangle_{A_1B_1}|\Phi_{00}\rangle_{A_2B_2}\cdots|\Phi_{00}\rangle_{A_nB_n}. \tag{10}$$

Alice performs 2-qudit Φ-state projective measurements on the qudit pairs (X_1, A_1), (X_2, A_2), $\cdots$, (X_2, A_2), respectively. The 2-qudit Φ-state set $\{|\Phi_{kl}\rangle_{AB} = U_A^{(kl)}|\Phi_{00}\rangle_{AB} = V_B^{(kl)}|\Phi_{00}\rangle_{AB};\ k, l \in \{0, 1, \cdots, d-1\}\}$ is a complete orthonormal basis set in d^2 dimensional Hilbert space for two qudits, where

$$U^{(kl)} = \sum_{j=0}^{d-1} e^{-2\pi i\overline{|j-l|}k/d}|\overline{|j-l|}\rangle\langle j|/\sqrt{d}, \tag{11}$$

$$V^{(kl)} = \sum_{j=0}^{d-1} e^{2\pi ijk/d}|\overline{j+l}\rangle\langle j|/\sqrt{d}, \tag{12}$$

$$\overline{j+l} = (j+l) \bmod d. \tag{13}$$

Incidentally, since Φ-states can be transformed into each other via the local unitary operations, the quantum channel linking Alice and Bob can also be other TPSs such as $|\Theta_{k_1l_1k_2l_2\cdots k_nl_n}\rangle_{A_1B_1A_2B_2\cdots A_nB_n}$ instead of the TPS $|\Theta_{0000\cdots00}\rangle_{A_1B_1A_2B_2\cdots A_nB_n}$, where

$$\begin{aligned} |\Theta_{k_1l_1k_2l_2\cdots k_nl_n}\rangle_{A_1B_1A_2B_2\cdots A_nB_n} &= U_{A_1}^{(k_1l_1)}U_{A_2}^{(k_2l_2)}\cdots U_{A_n}^{(k_nl_n)}|\Theta_{0000\cdots00}\rangle_{A_1B_1A_2B_2\cdots A_nB_n} \\ &= V_{B_1}^{(k_1l_1)}V_{B_2}^{(k_2l_2)}\cdots V_{B_n}^{(k_nl_n)}|\Theta_{0000\cdots00}\rangle_{A_1B_1A_2B_2\cdots A_nB_n}. \end{aligned} \tag{14}$$

After Alice's measurements, the system's state collapses to

$$\begin{aligned} &(|\Theta_{k_1l_1k_2l_2\cdots k_nl_n}\rangle_{A_1X_1A_2X_2\cdots A_nX_n}\langle\Theta_{k_1l_1k_2l_2\cdots k_nl_n}|)|\Gamma\rangle_{X_1X_2\cdots X_nA_1B_1A_2B_2\cdots A_nB_n} \\ =\ &(|\Phi_{k_1l_1}\rangle_{A_1X_1}\langle\Phi_{k_1l_1}|)(|\Phi_{k_2l_2}\rangle_{A_2X_2}\langle\Phi_{k_2l_2}|)\cdots(|\Phi_{k_nl_n}\rangle_{A_nX_n}\langle\Phi_{k_nl_n}|)|\Gamma\rangle_{X_1X_2\cdots X_nA_1B_1A_2B_2\cdots A_nB_n} \\ =\ &|\Phi_{k_1l_1}\rangle_{A_1X_1}|\Phi_{k_2l_2}\rangle_{A_2X_2}\cdots|\Phi_{k_nl_n}\rangle_{A_nX_n}({}_{A_1X_1}\langle\Phi_{k_1l_1}|{}_{A_2X_2}\langle\Phi_{k_2l_2}|\cdots{}_{A_nX_n}\langle\Phi_{k_nl_n}|)|\Gamma\rangle_{X_1X_2A_1B_1A_2B_2\cdots A_nB_n} \\ =\ &|\Phi_{k_1l_1}\rangle_{A_1X_1}|\Phi_{k_2l_2}\rangle_{A_2X_2}\cdots|\Phi_{k_nl_n}\rangle_{A_nX_n}({}_{A_1X_1}\langle\Phi_{00}|U_{A_1}^{(k_1l_1)\dagger})({}_{A_2X_2}\langle\Phi_{00}|U_{A_2}^{(k_2l_2)\dagger})\cdots({}_{A_nX_n}\langle\Phi_{00}|U_{A_n}^{(k_nl_n)\dagger}) \\ &\times|\Lambda\rangle_{X_1X_2\cdots X_n}|\Phi_{00}\rangle_{A_1B_1}|\Phi_{00}\rangle_{A_2B_2}\cdots|\Phi_{00}\rangle_{A_nB_n} \\ =\ &|\Phi_{k_1l_1}\rangle_{A_1X_1}|\Phi_{k_2l_2}\rangle_{A_2X_2}\cdots|\Phi_{k_nl_n}\rangle_{A_nX_n}({}_{A_1X_1}\langle\Phi_{00}|{}_{A_2X_2}\langle\Phi_{00}|\cdots{}_{A_nX_n}\langle\Phi_{00}|) \\ &\times|\Lambda\rangle_{X_1X_2\cdots X_n}V_{B_1}^{(k_1l_1)\dagger}|\Phi_{00}\rangle_{A_1B_1}V_{B_2}^{(k_2l_2)\dagger}|\Phi_{00}\rangle_{A_2B_2}\cdots V_{B_n}^{(k_nl_n)\dagger}|\Phi_{00}\rangle_{A_nB_n} \\ =\ &\frac{1}{d^n}|\Phi_{k_1l_1}\rangle_{A_1X_1}|\Phi_{k_2l_2}\rangle_{A_2X_2}\cdots|\Phi_{k_nl_n}\rangle_{A_nX_n}V_{B_1}^{(k_1l_1)\dagger}V_{B_2}^{(k_2l_2)\dagger}\cdots V_{B_n}^{(k_nl_n)\dagger}|\Lambda\rangle_{B_1B_2\cdots B_n} \\ =\ &\frac{1}{d^n}|\Theta_{k_1l_1k_2l_2\cdots k_nl_n}\rangle_{A_1X_1A_2X_2\cdots A_nX_n}V_{B_1}^{(k_1l_1)\dagger}V_{B_2}^{(k_2l_2)\dagger}\cdots V_{B_n}^{(k_nl_n)\dagger}|\Lambda\rangle_{B_1B_2\cdots B_n}. \end{aligned} \tag{15}$$

This means that if Alice gets the state $|\Phi_{k_1l_1}\rangle_{A_1X_1}|\Phi_{k_2l_2}\rangle_{A_2X_2}\cdots|\Phi_{k_nl_n}\rangle_{A_nX_n}$ via her measurement, then the state of Bob's qudits $\{B_1, B_2, \cdots, B_n\}$ collapses to the state $V_{B_1}^{(k_1l_1)\dagger}V_{B_2}^{(k_2l_2)\dagger}\cdots V_{B_n}^{(k_nl_n)\dagger}|\Lambda\rangle_{B_1B_2\cdots B_n}$. Further, if Alice tells Bob her results (i.e., $(k_1l_1k_2l_2\cdots k_nl_n)$) via public channel, then Bob can recover the state $|\Lambda\rangle$ in his qudits $\{B_1, B_2, \cdots, B_n\}$ by performing the local unitary operations $V_{B_1}^{(k_1l_1)}$, $V_{B_2}^{(k_2l_2)}$, $\cdots$, and $V_{B_n}^{(k_nl_n)}$, respectively. Up to now, we have presented the protocol $\mathcal{D}_n$ for teleporting arbitrary

n-qudit state using TPS of n GBSs. By the way, when the dimensionality d of the qudit state in our protocol $\mathcal{D}_n$ is 2, then the present protocol becomes the Yang-Guo protocol[4]. Further, if n is equal to 2, then the present protocol becomes the Lee-Min-Oh protocol in Ref.[3].

III. Protocols $\mathcal{D}'_n$ and $\mathcal{D}''_n$ using two classes of GESs

In the last section we have shown a protocol $\mathcal{D}_n$ for teleporting arbitrary n-qudit state using TPS of n GBSs. Now we consider the teleportation of the same n-qudit state using another entangled quantum channel between Alice and Bob as follows,

$$|\Xi_{0000\cdots00}\rangle_{A_1B_1A_2B_2\cdots A_nB_n} = \Upsilon_{A_1A_2\cdots A_n}|\Theta_{0000\cdots00}\rangle_{A_1B_1A_2B_2\cdots A_nB_n}. \tag{16}$$

where $\Upsilon_{A_1A_2\cdots A_n}$ is a global unitary operator acting on the n qudits $A_1, A_2, \cdots, A_n$ and can not be reducible to n local operators acting the n qudits. The state of the $3n$ qudits X_1,X_2,$\cdots$,X_n,A_1,B_1,A_2,B_2, $\cdots$, A_n, B_n is

$$|\Gamma'\rangle_{X_1X_2\cdots X_nA_1B_1A_2B_2\cdots A_nB_n} = |\Lambda\rangle_{X_1X_2\cdots X_n}|\Xi_{0000\cdots00}\rangle_{A_1B_1A_2B_2\cdots A_nB_n}. \tag{17}$$

The $2n$-qudit state set $\{|\Xi_{k_1l_1k_2l_2\cdots k_nl_n}\rangle_{A_1B_1A_2B_2\cdots A_nB_n} = \Upsilon_{A_1A_2\cdots A_n}|\Theta_{k_1l_1k_2l_2\cdots k_nl_n}\rangle_{A_1B_1A_2B_2\cdots A_nB_n};$ $k_x, l_x \in \{0, 1, \cdots d\}\}$ is another complete orthonormal basis set for $2n$ qudits. Different Ξ states can be transformed into each other via local unitary operations. Hence, other Ξ states can also be used as the quantum channel instead of the state $|\Xi_{0000\cdots00}\rangle_{A_1B_1A_2B_2\cdots A_nB_n}$. Alice performs the Ξ-state projective measurement on the qubits $X_1, X_2, \cdots, X_n, A_1, A_2, \cdots, A_n$ in her site,

$$\begin{aligned}
&|\Xi_{k_1l_1k_2l_2\cdots k_nl_n}\rangle_{A_1X_1A_2X_2\cdots A_nX_n}\langle\Xi_{k_1l_1k_2l_2\cdots k_nl_n}|\Gamma'\rangle_{X_1X_2\cdots X_nA_1B_1A_2B_2\cdots A_nB_n} \\
&= |\Xi_{k_1l_1k_2l_2\cdots k_nl_n}\rangle_{A_1X_1A_2X_2\cdots A_nX_n}({}_{A_1X_1A_2X_2\cdots A_nX_n}\langle\Theta_{k_1l_1k_2l_2\cdots k_nl_n}|\Upsilon^\dagger_{A_1A_2\cdots A_n} \\
&\times|\Lambda\rangle_{X_1X_2\cdots X_n}\Upsilon_{A_1A_2\cdots A_n}|\Theta_{0000\cdots00}\rangle_{A_1B_1A_2B_2\cdots A_nB_n} \\
&= \frac{1}{d^n}|\Xi_{k_1l_1k_2l_2\cdots k_nl_n}\rangle_{A_1X_1A_2X_2\cdots A_nX_n}V^{(k_1l_1)\dagger}_{B_1}V^{(k_2l_2)\dagger}_{B_2}\cdots V^{(k_nl_n)\dagger}_{B_n}|\Lambda\rangle_{B_1B_2\cdots B_n}.
\end{aligned} \tag{18}$$

This indicates that if Alice obtains the state $|\Xi_{k_1l_1k_2l_2\cdots k_nl_n}\rangle_{A_1X_1A_2X_2\cdots A_nX_n}$ via her measurement, then the state of Bob's n qudits $B_1, B_2, \cdots, B_n$ collapses to $V^{(k_1l_1)\dagger}_{B_1}V^{(k_2l_2)\dagger}_{B_2}\cdots V^{(k_nl_n)\dagger}_{B_n}|\Lambda\rangle_{B_1B_2\cdots B_n}$. Further, if Alice informs Bob of her results (i.e., $(k_1l_1k_2l_2\cdots k_nl_n)$) via public channel, then Bob can recover the state $|\Lambda\rangle$ in his n qudits $B_1, B_2, \cdots, B_n$ by performing the local unitary operations $V^{(k_1l_1)}_{B_1}$, $V^{(k_2l_2)}_{B_2}$, $\cdots$, and $V^{(k_nl_n)}_{B_n}$, respectively. Since $\Upsilon_{A_1A_2\cdots A_n}$ is a global unitary operator and can not be reducible to n local operators acting on the n qudits $A_1, A_2, \cdots, A_n$, the Ξ states can not be reducible to the Θ states. Hence, Ξ states is different from the TPSs of n GBSs and is referred to as a kind of genuine entangled states for it is also a candidates for teleporting an arbitrary n-qudit state.

So far, we have presented the protocol $\mathcal{D}'_n$ for teleporting arbitrary n-qudit state using a class of GESs. In the special case of $n = 2$, $d = 2$ and $\Upsilon = \cos\theta_{12}|00\rangle\langle00| + \sin\theta_{12}|11\rangle\langle00| - \sin\theta_{12}|00\rangle\langle11| + \cos\theta_{12}|11\rangle\langle11| - \sin\phi_{12}|01\rangle\langle01| + \cos\phi_{12}|10\rangle\langle01| + \cos\phi_{12}|01\rangle\langle10| + \sin\phi_{12}|10\rangle\langle10|$, the present protocol $\mathcal{D}'_n$ is exactly the Yeo-Chua protocol in Ref.[8]. By the way, if $\Upsilon_{A_1A_2\cdots A_n}$ can be reducible to n local operators acting on the n qudits $A_1, A_2, \cdots, A_n$, then the protocol $\mathcal{D}'_n$ is transformed into the protocol $\mathcal{D}_n$.

Now let us present our protocol $\mathcal{D}''_n$ for teleporting an arbitrary n-qudit state using another class of GESs as quantum channel. The entangled quantum channel between Alice and Bob is as follows,

$$|\Xi'_{0000\cdots00}\rangle_{A_1B_1A_2B_2\cdots A_nB_n} = \Upsilon_{A_1A_2\cdots A_n}\Omega_{B_1B_2\cdots B_n}|\Theta_{0000\cdots00}\rangle_{A_1B_1A_2B_2\cdots A_nB_n}. \tag{19}$$

where $\Omega_{B_1B_2\cdots B_n}$ is a global unitary operator acting on the n qudits $B_1, B_2, \cdots, B_n$ and can not be reducible to n local operators acting the n qudits. Obviously, this entangled quantum channel

$|\Xi'_{0000\cdots 00}\rangle_{A_1B_1A_2B_2\cdots A_nB_n}$ is different from $|\Xi_{0000\cdots 00}\rangle_{A_1B_1A_2B_2\cdots A_nB_n}$ used in the protocol $\mathcal{D}'_n$. However, since the n qudits $B_1, B_2, \cdots, B_n$ are in Bob's site, before teleportation he can perform the unitary operation $\Omega^{\dagger}_{B_1B_2\cdots B_n}$. After his performance, the quantum channel is transformed into the first class of GESs. Surely, the teleportation can be realized. Hence the protocol $\mathcal{D}''_n$ is only a slight variation of the protocol $\mathcal{D}'_n$ but using different quantum channels. One can easily see that, the Chen-Zhu-Guo protocol is only our present protocol $\mathcal{D}''_n$ in the special case of $d = 2$. By the way, if $\Omega_{B_1B_2\cdots B_n}$ can be reducible to n local operators acting the n qudits $B_1, B_2, \cdots, B_n$, then the protocol $\mathcal{D}''_n$ is transformed into the protocol $\mathcal{D}'_n$ with other Ξ state as quantum channel.

4 Summary

To summarize, in this paper we have presented a protocol $\mathcal{D}_n$ for faithfully teleporting an arbitrary n-qudit state with the TPS of n GBSs as the quantum channel. Moreover, we have also put forward explicit protocols $\mathcal{D}'_n$ and $\mathcal{D}''_n$ for faithfully teleporting an arbitrary n-qudit state with two classes of $2n$-qudit GESs as the quantum channel, respectively.

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References

[1] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W. K. Wotters, Phys. Rev. Lett. **70**, 1895 (1993).
[2] M. A. Nielsen and I. L. Chuang, Quantum Computation and Quantum Information (Cambridge University Press, Cambridge, 2000).
[3] J. Lee, H. Min, and S. D. Oh, Phys. Rev. A 66, 052318 (2002).
[4] C. P. Yang and G. C. Guo, Chin. Phys. Lett. 17, 162 (2000).
[5] Z. J. Zhang, Phys. Lett. A 351, 55 (2006).
[6] G. Rigolin, Phys. Rev. A 71, 032303 (2005).
[7] F. G. Deng, Phys. Rev.A 72, 036301 (2005).
[8] Y. Yeo and W. K. Chua, Phys. Rev. Lett. 94, 060502 (2006).
[9] P. X. Chen, S. Y. Zhu and G. C. Guo, Phys. Rev. A 74, 032324 (2006).
[10] M. S. Zubairy, Phys. Rev. A 58, 4368 (1998).
[11] S. Stenholm, P. J. Bardroff, Phys. Rev. A 58, 4373 (1998).
[12] L. Roa, A. Delgado, and I. Fuentes-Guridi, Phys. Rev. A 68, 022310 (2003).